MATHEMATICS 264 Fall 2006 Assignment 2

Due in class Thursday Sept. 28

1. Find the volume in the upper half space $z \geq 0$ below the cone $x^2+y^2 = z^2$ inside the cylinder $x^2 + y^2 = 2x$.

2. Find the area of the surface of the cone $x^2 + y^2 = z^2$ which lies in the upper half space $z \geq 0$ and inside the cylinder $x^2 + y^2 = 2y$.

3. Using spherical polar co-ordinates, calculate

$$\overline{z} = \frac{\int\int\int_{\mathcal{R}} z \mathrm{d}v}{\int\int\int_{\mathcal{R}} \mathrm{d}V}$$

   where dV is the element of volume and $\mathcal{R}$ is the region in the upper half space $z \geq 0$ inside the sphere of radius $a > 0$ centered at the origin which lies above the cylinder $x^2 + y^2 = z^2$

4. For the surface $\mathbf{r}(u, v)$, the vector element of surface is given by

$$\mathrm{d}\mathbf{S} = \mathbf{r}_u \times \mathbf{r}_v \mathrm{d}u\mathrm{d}v = \mathbf{n}\mathrm{d}S$$

   where $\mathbf{n}$ is the unit normal to the tangent plane. With this convention, calculate, for the hemisphere $\mathcal{R}$ of radius $a > 0$ centered at the origin with $z \geq 0$, using spherical polar co-ordinates (take $u = \theta, v = \phi$)

   (a) $\int\int_{\mathcal{R}} \mathbf{r} \cdot \mathbf{n}\mathrm{d}S$

   (b) $\int\int_{\mathcal{R}} ||\mathbf{r}||^{-3}\mathbf{r} \cdot \mathbf{n}\mathrm{d}S$