Mathematics 264, Assignment 3

Due in class Thursday Oct. 12

1. Compute the following three line integrals directly around the boundary Γ of the part $\mathcal{R}$ of the interior ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1, a > 0, b > 0$ that lies in the first quadrant:

   (a) $\oint_\Gamma x\mathrm{d}y - y\mathrm{d}x$

   (b) $\oint_\Gamma x^2\mathrm{d}y$

   (c) $\oint_\Gamma y^2\mathrm{d}x$

2. Use Green's theorem to reinterpret these integrals as double integrals, then compute the x and y co-ordinates of the centroid of $\mathcal{R}$ using these results. Note: the parameterisation

$$x = a\cos\phi, \quad y = b\sin\phi$$

   over a suitable interval could be useful in parameterising the arc of the ellipse.

3. (a) for each of the following functions, compute both the Gradient and the Laplacian

      i. $f(x, y, z) = xy + yz + xz + \cos(xyz)$

      ii. $f(x, y, z) = e^{xy}\cos z + \tan yz \sin xz$

   (b) for each of the following vector valued functions, compute the Divergence and the Curl

      i. $\mathbf{F} = (x^2 + y^2)\mathbf{i} + x\cos yz\mathbf{j} + e^{x^3+y^3}\mathbf{k}$

      ii. $\mathbf{F} = (x^2y^2)\mathbf{i} + x^2\sin yz\mathbf{j} + e^{x^3y^3}\mathbf{k}$

4. For the region $\mathcal{R}$ in the first octant bounded above by the plane $x+y+z = 1$ and the surface $\mathcal{S}$ which is the boundary of this region, calculate, for the vector valued function $\mathbf{F} = (x^2 + y^2)\mathbf{i} + (y^2 + z^2)\mathbf{j} + (z^2 + x^2)\mathbf{k}$:

   (a) $\int\int\int_\mathcal{R} \mathrm{div}\mathbf{F}\mathrm{d}V$

   (b) $\int\int_\mathcal{S} \mathbf{F}\cdot\mathbf{n}\mathrm{d}\mathcal{S}$

   Note: $\mathbf{n}$ is the outward pointing vector.