Mathematics 264 Assignment 4

Due on Thursday October 26, 2006 in class

(1) (a) Verify that the vector field $\mathbf{F} = (1 + yz)\mathbf{i} + (2y + xz)\mathbf{j} + (3z^2 + xy)\mathbf{k}$ is conservative by first showing $\nabla \times \mathbf{F} = 0$, then finding a potential $\phi(x, y, z)$ such that $\mathbf{F} = \nabla\phi$

(b) Find the flux of $\mathbf{F} = 2x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$ upward through the surface

$$(S): \quad \mathbf{r} = u^2v\mathbf{i} + uv^2\mathbf{j} + v^3\mathbf{k}, \quad (0 \leq u \leq 1; 0 \leq v \leq 1).$$

(2) Consider the vector field

$$\mathbf{F} = \mathbf{r} + \nabla\left(\frac{1}{||\mathbf{r}||}\right).$$

Compute the flux $\int\int_S \mathbf{F} \cdot \mathbf{n} dS$ of $\mathbf{F}$ across the surface of the sphere $x^2 + y^2 + z^2 = a^2, a > 0$. As usual, $\mathbf{n} dS$ is the vector element of surface with $\mathbf{n}$ the unit normal which here is assumed to point away from the enclosed volume.

(3) Let S be the subset of the surface of the sphere $x^2 + y^2 + z^2 = 9$ for which $x^2 + y^2 \geq 2$, and let $\mathbf{F}$ be the vector field defined by

$$\mathbf{F} = (-y, x, xyz).$$

Compute

$$\int\int_S (\nabla \times \mathbf{F}) \cdot \mathbf{n}\, dS,$$

where S is oriented so that the unit normal $\mathbf{n}$ to S points away from the enclosed volume.

(4) For the surface S (helicoid or spiral ramp) swept out by the line segment joining the point $(2t, \cos t, \sin t)$ to $(2t, 0, 0)$ where $0 \leq t \leq \pi$,

(a) Find a parametrisation for this surface S and of the boundary γ of this surface.

(b) For the vector field $\mathbf{F} = (x, y, z)$ compute the flux of $\mathbf{F}$ through the surface of part (a). Assume the normal to the surface has a non-negative $\mathbf{k}$ component at $t = 0$.

(c) Compute $\int_\gamma \mathbf{F} \cdot d\mathbf{r}$ where γ is the boundary curve of the above surface and $\mathbf{F}$ the above mentioned force field.