Mathematics 264, Assignment 6

Due on Tuesday November 21, 2006 in class

(1) Given a uniform one-dimensional rod with heat sources $F = \frac{q}{C\rho} = x$. Assume that the thermal diffusivity of the rod is $\kappa_{\mathrm{T}} = 2$ and thermal conductivity of rod is $k_0 = 1.5$, its ends are located at $x = 0$ and $x = 10$ and kept at 150 and 100, respectively. Determine

(a) the steady state of temperature distribution in a bar.

(b) the heat flux at each of the end points.

(2) Solve each of the following boundary/initial value problems by the method of separation of variables:

(a)
$$\frac{\partial u}{\partial t} = \frac{\partial u}{\partial x} - 2u, \quad u(x,0) = 10\mathrm{e}^{-x} - 6\mathrm{e}^{-4x}.$$

(b)
$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + 2u, \ (0 < x < 4), \qquad u(0,t) = u(4,t) = 0, \ u(x,0) = 6\sin\frac{\pi x}{2} + 3\sin\pi x.$$

(3) (a) Expand the function
$$f(x) = \begin{cases} x & (0 < x < 4); \\ 8 - x & (4 < x < 8) \end{cases}$$
(i) in a Fourier sine series; (ii) in a Fourier cone series.

(b) Expand the function $f(x) = \cos x, \ 0 < x < \pi$ in a Fourier sine series.

(4) Solve the following problems by using the Fourier series:

(a)
$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \ (-\pi < x < \pi), \qquad u(-\pi,t) = u(\pi,t) = 0, \ u(x,0) = x^2$$

(b)
$$\frac{\partial u}{\partial t} = 4\frac{\partial^2 u}{\partial x^2}, \ (-\pi < x < \pi), \qquad u_x(-\pi,t) = u_x(\pi,t) = 0, \ u(x,0) = 2x$$

(c)
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial^2 u}{\partial x^2}, \ (0 < x < \pi), \qquad u(0,t) = u(\pi,t) = 0, \ u(x,0) = 0, u_t(x,0) = f(x)$$
where
$$f(x) = \begin{cases} x & (0 < x < \frac{\pi}{2}); \\ \frac{\pi}{2} - x & (\frac{\pi}{2} < x < \pi) \end{cases}$$