Department of Mathematics and Statistics, McGill University
MATH 265 ADVANCED CALCULUS: ASSIGNMENT 1

This assignment is due in class on Tuesday, January 27, 2004.

1. A function u of 2 variables is said to be *harmonic* if $\bigtriangledown^2 u = u_{11} + u_{22} = 0$.

   (a) (Adams 12.5.16) Show that if $u(x, y)$ is harmonic, then so is

   $$v(x, y) = u(\frac{x}{x^2 + y^2}, \frac{-y}{x^2 + y^2})$$

   .

   (b) (Adams 12.5.25) Let $x = e^s \cos t,\ y = e^s \sin t$ and $u(x, y) = v(s, t)$. Show that $u_{ss} + u_{tt} = (x^2 + y^2)(v_{xx} + v_{yy})$, from which it follows that u is harmonic if and only if v is harmonic.

2. Evaluate $\frac{\partial}{\partial \alpha} \int_{\alpha}^{\alpha^2} (x^2 + \alpha^3)\, dx$ in two different ways, by first integrating and by first differentiating.

3. Verify that $y(t) = e^t + \displaystyle\int_0^t \sinh(t - r) g(r)\, dr$ satisfies the differential equation $y''(t) - y = g(t)$. What are the initial values of $y(0)$ and $y'(0)$?

4. Use the Taylor expansion to find

   (a) the linear approximation to $1 + x^2 + \ln(1 + x + y)$ at $(1, 1/2)$;

   (b) the quadratic approximation to $e^{(x-y)} + \cos(2x + 3y)$ at $(0, 0)$.

5. (Adams 12.6.16) Find the Jacobian matrix $D\mathbf{g}(1, 3, 3)$ for the transformation of $\mathbb{R}^3$ to $\mathbb{R}^3$ given by $\mathbf{g}(r, s, t) = (r^2 s, r^2 t, s^2 - t^2)$ and use this to find an approximate value for $\mathbf{g}(0.99, 3.02, 2.97)$.

6. Show that the equations

   $$\begin{aligned} xy - xu^3 + y^2 v + u &= -1, \\ xv - x^2 yu &= -8 \end{aligned}$$

   determine x and y as functions of u and v when (x, y, u, v) is close to $(2, 1, 1, -2)$. Find $\begin{pmatrix} x_u & x_v \\ y_u & y_v \end{pmatrix}$ when $x = 2, y = 1, u = 1, v = -2$.

7. (Adams 12.8.18) Show that the equations

   $$\begin{aligned} xe^y + uz - \cos v &= 2, \\ u \cos y + x^2 v - yz^2 &= 1 \end{aligned}$$

   determine u and v as functions of x, y and z near the point where $(x, y, z) = (2, 0, 1)$ and $(u, v) = (1, 0)$. Find u_z at $(2, 0, 1)$.

8. Use the method of Lagrange multipliers to find the maximum and minimum values of the function xyz on the curve of intersection of $x^2+y^2+z^2 = 6$ and $x^2+2x+y^2+z^2 = 8$.

January 2, 2004.