Department of Mathematics and Statistics, McGill University
MATH 265 ADVANCED CALCULUS: ASSIGNMENT 2

This assignment is due in class on Tuesday, February 10, 2004.

1. Evaluate $\iint_D (x^4 - y^4)\, dxdy$ where D is the region in the first quadrant with
$$1 \le x^2 - y^2 \le 3, \quad 2 \le xy \le 3.$$

2. Evaluate $\iiint_D x\, dxdydz$ where D is the region described by
$$1 \le x + y \le 4, \quad -1 \le 2x - y \le 3, \; -1 \le y + z \le 2.$$

3. Find the coordinates of the centroid of the region inside the sphere $x^2 + y^2 + z^2 = a^2$, outside the cone $x^2 + y^2 = b^2 z^2$, and above the xy-plane, where $a > 0, \; b > 0$.

4. Find the coordinates of the centroid of the paraboloidal surface
$z = 4 - x^2 - y^2$, above the xy-plane.

5. Find the field lines for $\mathbf{F}(x, y) = y\mathbf{i} + x\mathbf{j}$ and show this is a conservative vector field by finding a potential function. Sketch the equipotential curves and the field lines.

6. (Adams 15.2.10) By finding a suitable potential show that the vector field
$$\mathbf{F}(x, y, z) = -\frac{y^2 + 2z^2}{x^2}\mathbf{i} + \frac{2y}{x}\mathbf{j} + \frac{4z}{x}\mathbf{k}$$
is conservative. Describe the equipotential surfaces and find parametric representations of the field lines.

7. (Adams 15.3.8) Find $\int_{\mathcal{C}} \sqrt{1 + 4x^2 z^2}\, ds$, where $\mathcal{C}$ is the curve of intersection of the surfaces $x^2 + z^2 = 1$ and $y = x^2$.

8. (Adams 15.3.10) Find the mass and centre of mass of a piece of wire bent in the shape of a circular helix $x = \cos t, \; y = \sin t, \; z = t$ with $0 < t < \pi$ if the wire has density given by $\delta(x, y, z) = z$.

January 25, 2004.