Department of Mathematics and Statistics, McGill University
MATH 265 ADVANCED CALCULUS: ASSIGNMENT 3

This assignment is due in class on Thursday, February 19, 2004 but will, *exceptionally* also be accepted as late as Friday, February 20, by the instructor of the class you attend. You may wish to keep a photocopy with which to study for the examination. Solutions will be posted on the weekend.

1. Consider the vector field $\mathbf{F} = (2x + y)\mathbf{i} + (3x - 2y)\mathbf{j}$. Evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where $\mathcal{C}$ is
   (a) the straight line segment from $(0,0)$ to $(1,1)$;
   (b) the parabola $y = x^2$ from $(0,0)$ to $(1,1)$;
   (c) the curve $y = \sin(\pi x/2)$ from $(0,0)$ to $(1,1)$;
   (d) the curve $y = x^n$ $(n > 0)$ from $(0,0)$ to $(1,1)$.

2. A particle is moved counterclockwise around the square $0 \le x \le 1$, $0 \le y \le 1$, $z = 0$ under the action of the force field $\mathbf{F} = (x - y^2)\mathbf{i} + (2y + x^2)\mathbf{j} + x\mathbf{k}$. Calculate the work done.

3. Consider the vector field $\mathbf{F} = (x - z)\mathbf{i} + (y - z)\mathbf{j} - (x + y)\mathbf{k}$.
   (a) Evaluate the integral of the tangential component of $\mathbf{F}$ along the polygonal path from $(0,0,0)$ to $(1,0,0)$ to $(1,1,0)$ to $(1,1,1)$.
   (b) Now evaluate the integral of the tangential component of $\mathbf{F}$ along the line segment from $(0,0,0)$ to $(1,1,1)$.
   (c) You should have found the same value in parts (a) and (b). Why? Give an alternative way of obtaining the value you found before.

4. (Adams 15.4.10) Find constants a and b such that the field $\mathbf{F}(x,y,z) = (axy + z)\mathbf{i} + x^2\mathbf{j} + (bx + 2z)\mathbf{k}$ is conservative and determine the corresponding potential. Then, using these constants, evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where $\mathcal{C}$ is the curve from $(1,1,0)$ to $(0,0,3)$ that lies on the intersection of the surfaces $2x + y + z = 3$ and $9x^2 + 9y^2 + 2z^2 = 18$ in the positive octant.

5. Suppose that a certain frictional force is constant in magnitude and always acts in a direction which opposes any motion. Verify that the work done against friction in moving a body from one location to another is proportional to the length of the path.

6. (Adams 15.5.4) Find the area of that part of the sphere $x^2 + y^2 + z^2 = 4a^2$ which lies inside the cylinder $x^2 + y^2 = 2ay$ (with $a > 0$).

7. (Adams 5.5.20) Describe the parametric surface

$$x = au\cos v, \quad y = au\sin v, z = bv$$

   for $0 \le u \le 1,\ 0 \le v \le 2\pi$) (with $a, b > 0$). Find the area of that surface.

8. (variation on Adams 5.5.23 and 28) Find the mass and the centre of mass of a right circular conic shell of base radius a, height h and constant areal density δ. Then evaluate the moment of inertia about the axis of the shell.

9. If $n > -1$ evaluate $\iint_S y^n dS$ over the hemisphere of $x^2 + y^2 + z^2 = 1$ with $y > 0$.

January 25, 2004.