MATH 264B Advanced Calculus, Winter 2006

Assignment 2, due in class on Thursday February 2, 2006

1. Find the volume of the region bounded by the spheres $x^2+y^2+z^2=1$, $x^2+y^2+z^2=16$, the cone $z^2=x^2+y^2$, and the planes $z=0$, $y=0$, $y=x$. (**Hint:** The region consists of two parts (in the 1st and 3rd octants) symmetrical to each other.)

2. Determine the integral curves of the vector field

$$\mathbf{F}(x,y,z) = -y\mathbf{i} + x\mathbf{j} + xy\mathbf{k}.$$

3. Show that the vector field

$$\mathbf{F}(x,y,z) = yz(2x+y+z)\mathbf{i} + xz(x+2y+z)\mathbf{j} + xy(x+y+2z)\mathbf{k}$$

is conservative and find its potential.

4. Find the equipotential surfaces of the conservative vector field

$$\mathbf{F}(x,y,z) = \frac{2}{(y+z)^{1/2}}\mathbf{i} - \frac{x}{(y+z)^{3/2}}\mathbf{j} - \frac{x}{(y+z)^{3/2}}\mathbf{k}.$$

5. Evaluate the line integral

$$\int_{\mathcal{C}} y^2 \, ds,$$

where $\mathcal{C}$ is the arc of the cycloid

$$x = a(t-\sin t),\ y = a(1-\cos t) \quad (a = \text{const})$$

between 0 and 2π.