MATH 264B Advanced Calculus, Winter 2006

Assignment 5, due in class on Thursday March 30, 2006

1. Consider the vector fields

$$\mathbf{F} = (y, -x, xyx),\ \mathbf{G} = (xz, -yz, -2).$$

Show that $\nabla \times \mathbf{F} = \mathbf{G}$ and compute the flux of $\mathbf{G}$ across the surface of the cylinder $x^2 + z^2 = 1$, $-2 \leq y \leq 2$, oriented with the outward pointing normal.

2. Let

$$\mathbf{F} = \nabla \times \left((x^2 + y^2 + z^2)(\mathbf{i} + \mathbf{j} + \mathbf{k})\right).$$

Let c be the curve of intersection of the upper unit hemisphere $x^2+y^2+z^2 = 1$, $z \geq 0$, with the cylinder $x^2 + y^2 = \frac{1}{4}$, oriented counterclockwise when looking down from the positive Z axis. Rewrite the line integral

$$\oint_c \mathbf{F} d\mathbf{r},$$

as a surface integral using Stokes' Theorem and evaluate the surface integral.

3. Let

$$\mathbf{F} := \nabla \left(\frac{1}{\sqrt{x^2 + y^2 + z^2}} + x^3 + y^3 + z^3 \right).$$

a) Compute the outward flux of $\mathbf{F}$ across the sphere $x^2 + y^2 + z^2 = a^2$, $a > 0$.

b) Compute the outward flux of $\mathbf{F}$ across the cube $1 \leq x \leq 2$, $1 \leq y \leq 2, 1 \leq z \leq 2$.