1. (a) Compute $\int_C yz\,dx - xz\,dy + xy\,dz$ if C is given by $x = t$, $y = e^t$ $z = e^{-t}$, t
    from 0 to 1.

    (b) Compute $\int (y^2 + 2xyz)dx + (2xy + x^2z - z^2)dy + (x^2y - 2yz)dz$ if C consists of the
    curve $x = \sin t$, $y = \cos^2 t$, $z = t^2$ from $t = 0$ to $t = \pi$, followed by the line segment
    from $(0, 1, \pi^2)$ to $(2, -1, 1)$.

2. Calculate $\int\int_D \dfrac{x + 2y}{3x - y}dx\,dy$ where D is the parallelogram $-2 \leq x+2y \leq 1$, $1 \leq 3x-y < 3$.

3. Show that the equations
$$xy - xu^3 + y^2v + u = -1$$
$$xv - x^2yu = -2$$
determine x and y as functions of u and v when (x, y, u, v) is close to $(2, 1, 1, -2)$. Find
$$\begin{pmatrix} \dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v} \\[2mm] \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v} \end{pmatrix}$$
when $x = 2, y = 1, u = 1, v = -2$.

4. Compute the flux of $\vec{F} = y\vec{i} + x\vec{j} + z\vec{k}$ upwards through the part of the surface $z = xy$
which lies inside the cylinder $x^2 + y^2 = 4$.

5. (a) Express the equations of the sphere $x^2 + y^2 + z^2 = 9$ and the hyperboloid
    $x^2 + y^2 - z^2 = 1$ in cylindrical coordinates.

    (b) Compute the flux of $\vec{F} = x\vec{i} + y\vec{j} - z\vec{k}$ outwards through the boundary of the solid
    W that lies inside the sphere and outside the hyperboloid of part (a).

6. Let C be the circle $x^2 + y^2 = a^2$, taken counterclockwise.

    (a) Compute $\int_C \dfrac{x}{x^2 + y^2}dx + \dfrac{y}{x^2 + y^2}dy$,

    (b) Compute $\int_C \dfrac{-y}{x^2 + y^2}dx + \dfrac{x}{x^2 + y^2}dy$.

    (c) If C_1 is the circle $(x - 2)^2 + (y - 3)^2 = 1$, taken clockwise, compute
    $$\int_{C_1} \dfrac{-y}{x^2 + y^2}dx + \dfrac{x}{x^2 + y^2}dy.$$

7. Let C be the intersection of the cylinder $x^2 + y^2 = 9$ with the plane $x + y + z = 1$,
taken counterclockwise when looking down from above. Use Stokes' theorem to compute
$\int_C (2y + z)dx - (3x + z)dy + (3x - 2y)dz.$

McGILL UNIVERSITY

FACULTY OF ENGINEERING

FINAL EXAMINATION

MATHEMATICS 189-265B

ADVANCED CALCULUS

Examiner: Professor D. Sussman
Associate Examiner: Dr. P. Bracken

Date: Friday, April 30, 1999
Time: 2:00 P.M. - 5:00 P.M.

INSTRUCTIONS

Calculators are NOT permitted.

This exam comprises the cover and 1 page of questions.