1. (4 parts, 14 marks) Consider the planar vector field $\mathbf{F}(x,y) = 3x\mathbf{i} - y\mathbf{j}$.
1(a) (4 marks) Verify that $\mathbf{F}(x,y)$ is conservative, find equations for the equipotential curves and sketch them roughly.

1(b) (4 marks) Find the flow lines of $\mathbf{F}(x,y)$ and sketch them roughly.

1(c) (2 marks) Verify explicitly that the flow lines and the equipotential curves of $\mathbf{F}(x,y)$ indeed intersect orthogonally.

1(d) (4 marks) Verify that there is a flow line passing through both the points $(8,2)$ and $(1,4)$. Then evaluate $\int_{\mathcal{C}} \mathbf{F} \cdot d\mathbf{r}$, where $\mathcal{C}$ is the segment of the flow line passing from $(8,2)$ to $(1,4)$.

2. (3 parts, 13 marks) Consider the pair of equations

$$x^2 + y^2 - r^2 - 2s - 6 = 0, \quad x^3 + y^3 - r^3 + 3s - 68 = 0$$

2(a) (3 marks) Suppose that the values $(x, y, r, s) = (a, b, c, d)$ satisfy both equations. What condition has to be satisfied to guarantee that one can solve for x and y as functions of r and s satisfying $x(c, d) = a$ and $y(c, d) = b$?

2(b) (2 marks) Verify in particular that you can solve the equations to obtain functions $x(r, s)$ and $y(r, s)$ satisfying $x(4, 2) = 5$ and $y(4, 2) = 1$.

2(c) (8 marks) Find all first order partial derivatives of the functions $x(r, s)$ and $y(r, s)$ at $(r, s) = (4, 2)$ and find approximate values for $x(4.01, 1.99)$ and $y(4.01, 1.99)$ using linear approximation.

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3. (13 marks) Consider that part of the surface

$$x^2 + y^2 + (z + 5)^2 = 100$$

for which $x \geq 0$, $y \geq 0$, $z \geq 0$. Find the area and the coordinates of the centroid (i.e. the center of mass in case the surface has uniform density).